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GYROKINETIC LINEARIZED LANDAU COLLISION OPERATOR

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ABSTRACT

Here the full gyrokinetic electrostatic linearized Landau collision operator, entering the δF formulation of gyrokinetic theory, is calculated including the equilibrium operator $\mathcal{C}[F_{Ma}, F_{Mb}]$. Energy exchange between plasma species is described by $\mathcal{C}[F_{Ma}, F_{Mb}]$.

Furthermore, $\mathcal{C}[F_{Ma}, F_{Mb}]$ describes drag and diffusion of the magnetic field aligned component of the vorticity associated with the $\boldsymbol{E} \times \boldsymbol{B}$ -drift.

Contrary to conventional wisdom it is shown that $\mathcal{C}[F_{Ma}, F_{Ma}] \neq 0$ for **like-particle collisions** and has the same order of magnitude as the gyrokinetic test- and field-particle operators. $\mathcal{C}[F_{Ma}, F_{Ma}]$ **must be included in gyrokinetic δF collision operators.**

PARTICLES COLLIDE

Gyrokinetic theory decouples the fast time-scale associated with the fast cyclotron motion of charged particles in strongly magnetized plasmas from the low-frequency dynamics of gyro-centers. The gyrokinetic theory describes the time-evolution of gyro-centers.

A gyro-center is a mathematical construction. Gyro-centers do not carry charge nor do they collide. Therefore, caution must be exercised when describing particle dynamics in the gyro-center picture.

For instance in the gyrokinetic Gauss's Law the charge distribution must be deduced from the distribution of gyro-centers. Therefore, the gyro-center charge contribution is accompanied by polarization density and inverse gyro-angle averages, which in combination give the low-frequency particle charge distribution. **The polarization density originates from the Maxwellian (constant density) part of the gyrokinetic distribution function.**

Similarly, the Landau collision operator (and collisions in general) describes collisions between particles, not gyro-centers. Therefore, collision operators in gyrokinetic theory are expected to have terms similar to those in the gyrokinetic Gauss's Law. **A polarization-density-like contribution is expected in gyrokinetic collision operators**.

GYROKINETIC MAXWELLIAN

The Landau collision operator vanishes $\mathcal{C}[f_a, f_b] = 0$ when $f_{a,b}(\boldsymbol{x}, \boldsymbol{v})$ are Maxwellian $f_M = n_0(2\pi m T_0)^{-3/2}\exp(\frac{-mv^2}{2T_0})$, with the same temperature. However, when evaluating the collision operator with two gyrokinetic Maxwellians: $F_{Ma} = N(2\pi T m_a)^{-3/2}\exp(-\frac{1/2 m_a v_\parallel^2 + \mu B}{T})$ having the same temperature the Landau collision operator **does not vanish**:

$$\mathcal{C}[F_{Ma}, F_{Mb}] \neq 0.$$

f_M and F_M have the same functional form but do only agree to lowest order in ϵ:

$$F_M(\boldsymbol{Z}) = f_M(\boldsymbol{z})[1 - \epsilon q\tilde{\phi}(\boldsymbol{z})/T] + \mathcal{O}(\epsilon^2).$$

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LANDAU COLLISION OPERATOR

In this work we express the Landau collision operator in gyro-center coordinates and perform an explicit gyro-angle average. In $(\boldsymbol{x}, \boldsymbol{p})$ coordinates the Landau collision operator is

$$\mathcal{C}[f_a; f_b] = \frac{\partial}{\partial p_i}\left[\frac{m_a}{m_b} f_a \frac{\partial H_b}{\partial p_i} - m_a^2 \frac{\partial^2 G_b}{\partial p_i \partial p_j}\frac{\partial f_a}{\partial p_j}\right],$$

where the Rosenbluth potentials[1] are defined as:

$$\begin{pmatrix} H_b \\ G_b \end{pmatrix} = \frac{-\Gamma_{ab}}{8\pi}\int d^6 z'\, f_b(\boldsymbol{z}')\delta^3(\boldsymbol{x} - \boldsymbol{x}')\begin{pmatrix} 2u^{-1} \\ u \end{pmatrix}.$$

$u = |\boldsymbol{v} - \boldsymbol{v}'|$ is the relative velocity and $\Gamma_{ab} = (q_a q_b/\epsilon_0)^2 \ln\Lambda$.

The δF formulation arise when the gyro-angle averaged distribution function is split into an equilibrium part F_M and a perturbed part δF. The bi-linearity of the collision operator leads to:

$$\sum_{a,b}\mathcal{C}[\langle F_a\rangle; \langle F_b\rangle] = \sum_{a,b}\underbrace{\mathcal{C}[F_{Ma}; F_{Mb}]}_{\text{equilibrium-operator}} + \underbrace{\mathcal{C}[\delta F_a; F_{Mb}]}_{\text{test-particle-operator}} + \underbrace{\mathcal{C}[F_{Ma}; \delta F_b]}_{\text{field-particle operator}}$$

- *Main focus here*[2] is to calculate the gyro-angle average of $\mathcal{C}[F_{Ma}; F_{Mb}]$.
- The equilibrium operator is composed of *test-particle-like* and *field-particle-like* parts:

$$\mathcal{C}[F_{Ma}; F_{Mb}] = \mathcal{C}_M^T + \mathcal{C}_M^F$$

TEST-PARTICLE-LIKE OPERATOR

The gyro-angle average of $\mathcal{C}_M^T$ becomes:

$$\begin{aligned}\langle\mathcal{C}_M^T\rangle = \int d^3k\, e^{i\boldsymbol{k}\cdot\boldsymbol{X}}\Big\{&(1 - \frac{m_a}{m_b})\nu_s F_{aM}\left[\frac{m_a v_0^2}{T_a} + \frac{q_a\rho_0^2 k_\perp^2\phi_k}{2T_a}[J_0(k_\perp\rho_0) + J_2(k_\perp\rho_0)]\right] + \Gamma_{ab}\frac{m_a}{m_b}F_{Ma}F_{Mb}\\ &+ \nu_D F_{aM}\left[-\frac{m_a v_0^2}{T_a} - \frac{q_a\rho_0^2 k_\perp^2\phi_k}{2T_a}\{J_0(k_\perp\rho_0)[1 + \frac{2v_0^2}{c_\perp^2}] + \frac{J_2(k_\perp\rho_0)}{2}[1 - \frac{m_a v_0^2}{T_a}]\}\right]\\ &+ \nu_\parallel F_{aM}\left[\frac{m_a v_0^2}{2T_a}[\frac{m_a v_0^2}{T_a} - 1] - \frac{q_a\rho_0^2 k_\perp^2\phi_k}{2T_a}\Big(J_0(k_\perp\rho_0)[1 - \frac{m v_0^2}{T_a}] - \frac{J_2(k_\perp\rho_0)}{2}[1 + \frac{2m_a v_0^2}{T_a}]\Big)\right]\Big\}\end{aligned}$$

where $v_0^2 = v_\parallel^2 + 2\mu B/m_a$, and $\nu_s = (m_a^2 v_0)^{-1} dH_{0b}/dv_0$, $\nu_D = -2(m_a^2 v_0^3)^{-1} dG_{0b}/dv_0, \nu_\parallel = -2(m_a^2 v_0^2)^{-1} d^2 G_{0b}/dv_0^2$ denote the standard slowing-down, deflection and parallel velocity diffusion frequencies[3], respectively.

- The zeroth order terms (independent of ϕ) mainly describe energy exchange.

FIELD-LIKE-OPERATOR

The field-particle like operator is characterized by having the time-dependent electric potential residing in the Rosenbluth potentials. The gyro-angle average of the field-particle-like becomes:

$$\begin{aligned}\langle\mathcal{C}_M^F\rangle = \int d^3k\, e^{i\boldsymbol{k}\cdot\boldsymbol{X}}\sum_{n=-\infty}^{\infty} F_{Ma}J_{-n}(k_\perp\rho_0)\Big[&\Big(1 - \frac{m_a}{m_b}\Big)\Big(2\mu\frac{\partial}{\partial\mu} + v_\parallel\frac{\partial}{\partial v_\parallel}\Big)\frac{-q_b\phi_k h_{nk}}{m_a T_a}\\ &+ \Big(\frac{n^2 T_a}{4\mu B} + \frac{2\mu B - T_a}{2B}\frac{\partial}{\partial\mu} + \frac{\mu B(2\mu B - T_a)}{B^2}\frac{\partial^2}{\partial\mu^2} + \frac{2\mu B v_\parallel T_a}{B}\frac{\partial^2}{\partial\mu\partial v_\parallel} + \frac{m_a v_\parallel^2 - T_a}{2m_a}\frac{\partial^2}{\partial v_\parallel^2}\Big)\frac{2q_b\phi_k g_{nk})}{T_a^2}\Big],\end{aligned}$$

where

$$\begin{aligned}\begin{pmatrix} h_{nk} \\ g_{nk}\end{pmatrix} = -\frac{\Gamma_{ab}}{8\pi}\int d\mu' dv_\parallel' 2\pi\, F_{Mb}\Big[&[\delta_{n0} - J_0(k_\perp\rho_0')J_n(-k_\perp\rho_0')]\frac{\partial}{\partial\mu'}\\ &- \sum_{l=1}^{\infty}\frac{n}{2l}k_\perp\frac{\partial\rho_0'}{\partial\mu'}[J_{n+l}(-k_\perp\rho_0') - J_{n-l}(-k_\perp\rho_0')]\Big]\begin{pmatrix} 2u_n^{-1} \\ u_n\end{pmatrix}.\end{aligned}$$

Fortunately h_{nk} and g_{nk} are *time-independent*, and can be precomputed and re-used for all time-steps in numerical simulations.

- The ϕ-dependent terms in $\mathcal{C}_M^T$ and $\mathcal{C}_M^F$ describe viscous damping of the magnetic field aligned component of the $\boldsymbol{E} \times \boldsymbol{B}$ -vorticity: $\hat{\boldsymbol{b}} \cdot \nabla \times \boldsymbol{u}_E \simeq -k_\perp^2\phi_k/B$.
- In the gyrokinetic ordering $\mathcal{C}_M^T$ and $\mathcal{C}_M^F$ has the same order of magnitude as the standard test- and field-particle operators and must therefore be retained. Similar conclusions hold for model operators.

LIKE-PARTICLE COLLISIONS

Utilizing $\mathcal{C}[f_{Ma}, f_{Ma}] = 0$ allows us to evaluate the equilibrium operator:

$$\mathcal{C}[F_{Ma}, F_{Ma}] = \mathcal{C}[F_{Ma}\frac{q\langle\phi\rangle}{T}, F_{Ma}] + \mathcal{C}[F_{Ma}, F_{Ma}\frac{q\langle\phi\rangle}{T}]$$

using the gyrokinetic test- and field-particle operators[4, 5, 2]. $\mathcal{C}[F_{Ma}, F_{Ma}]$ has same order of magnitude as test- and field-particle operators. **Must be retained in order to describe ion-ion collisions correct.**